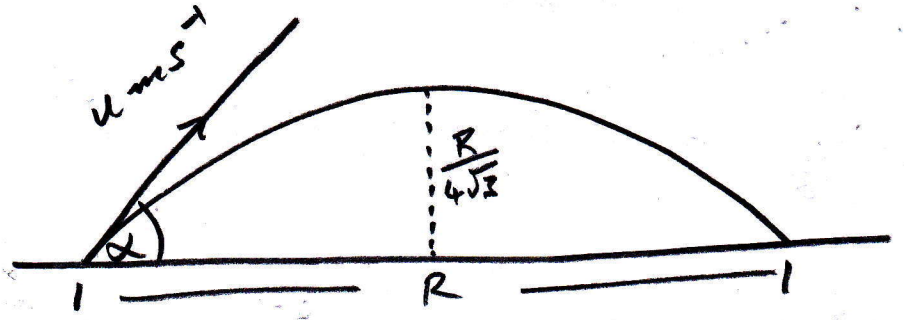
2013 Q3.

(a)



(i) $s_y = u_y t - \frac{1}{2} g t^2$

$\Rightarrow u \sin\alpha t - \frac{1}{2} g t^2 = 0$

$\Rightarrow t(u \sin\alpha - \frac{1}{2} g t) = 0$

$\Rightarrow t = 0$ or $u \sin\alpha - \frac{1}{2} g t = 0$

$\Rightarrow t = \frac{2u \sin\alpha}{g}$

$R = s_x = u_x t$

$= u \cos\alpha t$

$= u \cos\alpha \left(\frac{2u \sin\alpha}{g} \right)$

$= \frac{2u^2 \sin\alpha \cos\alpha}{g}$

(ii) Time $\frac{1}{2}$ way $= t_1 = \frac{u \sin\alpha}{g}$

$s_y = u_y t - \frac{1}{2} g t^2$

$\Rightarrow \frac{R}{4\sqrt{3}} = u \sin\alpha t_1 - \frac{1}{2} g t_1^2$

$\Rightarrow \frac{2u^2 \sin\alpha \cos\alpha}{4g\sqrt{3}} = u \sin\alpha \left(\frac{u \sin\alpha}{g}\right) - \frac{g}{2}\left(\frac{u \sin\alpha}{g}\right)^2$

$\times$ by g, $\div$ by $\sin\alpha$ and $\div$ by u^2

$\Rightarrow \frac{\cos\alpha}{2\sqrt{3}} = \sin\alpha - \frac{1}{2}\sin\alpha$

$\Rightarrow \frac{\cos\alpha}{2\sqrt{3}} = \sin\alpha\left(1 - \frac{1}{2}\right)$

$\Rightarrow \frac{\cos\alpha}{2\sqrt{3}} = \frac{1}{2}\sin\alpha$

$\Rightarrow \frac{2}{2\sqrt{3}} = \frac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \frac{1}{\sqrt{3}} = \tan\alpha$

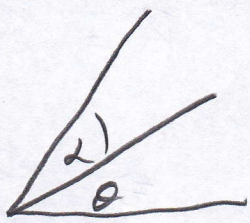
$\Rightarrow \alpha = \tan^{-1}\frac{1}{\sqrt{3}}$

$\Rightarrow \alpha = 30^\circ$

(b)

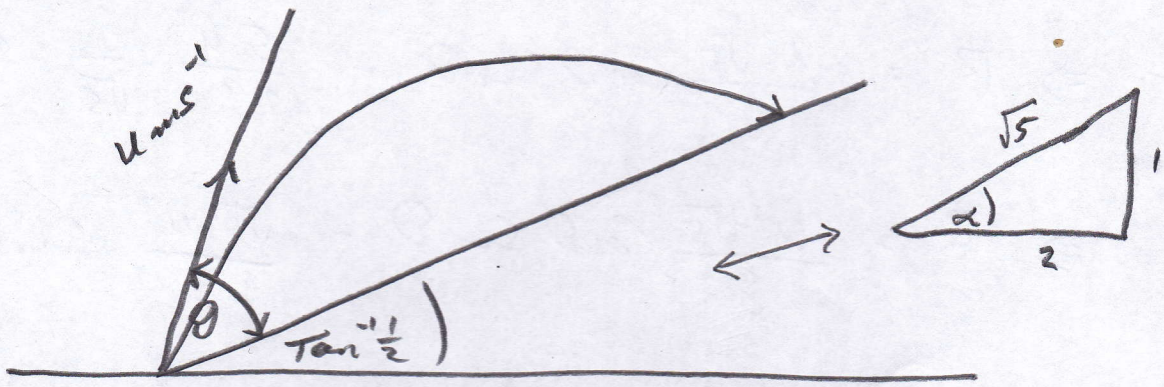
$$V_x = u\cos\alpha - g\sin\theta\, t$$

$$V_y = u\sin\alpha - g\cos\theta\, t$$



$$S_x = u\cos\alpha\, t - \tfrac{1}{2} g\sin\theta\, t^2$$

$$S_y = u\sin\alpha\, t - \tfrac{1}{2} g\cos\theta\, t^2$$



$S_y = 0$, find t:

$$S_y = u\sin\theta\, t - \tfrac{1}{2} g\cos\alpha\, t^2 = 0$$

$$\Rightarrow t\left(u\sin\theta - \tfrac{1}{2} g\cos\alpha\, t\right) = 0$$

$$\Rightarrow t = 0 \text{ or } u\sin\theta - \tfrac{1}{2} g\cos\alpha\, t = 0$$

$$\Rightarrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$$

$$\Rightarrow \quad t = \frac{2u\sin\theta}{g\cdot\frac{2}{\sqrt{5}}}$$

$$\Rightarrow \quad t = \frac{\sqrt{5}\,u\sin\theta}{g}$$

$$R = S_x = u\cos\theta t - \frac{1}{2}g\sin\alpha t^2$$

$$\Rightarrow R = u\cos\theta\left(\frac{\sqrt{5}u\sin\theta}{g}\right) - \frac{1}{2}g\sin\alpha\left(\frac{\sqrt{5}u\sin\theta}{g}\right)^2$$

$$\Rightarrow R = \frac{u^2\sqrt{5}}{g}\cos\theta\sin\theta - \frac{g5u^2}{2g^2}\sin\alpha\sin^2\theta$$

$$\Rightarrow R = \frac{u^2\sqrt{5}}{g}\cos\theta\sin\theta - \frac{5u^2}{2g}\cdot\frac{1}{\sqrt{5}}\sin^2\theta$$

$$\Rightarrow R = \frac{u^2\sqrt{5}}{g}\cos\theta\sin\theta - \frac{\sqrt{5}u^2}{2g}\sin^2\theta$$

$$\Rightarrow R = \frac{u^2\sqrt{5}}{g}\left(\cos\theta\sin\theta - \frac{1}{2}\sin^2\theta\right)$$

← [Factorise $\frac{1}{2}$

$$\Rightarrow R = \frac{u^2\sqrt{5}}{2g}\left(2\cos\theta\sin\theta - \sin^2\theta\right)$$

$$\Rightarrow R = \frac{u^2\sqrt{5}}{2g}\left(\sin 2\theta - \sin^2\theta\right)$$

$$\frac{dR}{d\theta} = \frac{u^2\sqrt{5}}{2g}\left(2\cos 2\theta - 2\sin\theta\cos\theta\right)$$

$$2\cos 2\theta - 2\sin\theta\cos\theta = 0$$

$$\Rightarrow 2\cos 2\theta = 2\sin\theta\cos\theta$$

$$\Rightarrow 2\cos 2\theta = \sin 2\theta$$

$$\Rightarrow 2 = \tan 2\theta$$

$$\Rightarrow 2\theta = 63.43^\circ \Rightarrow \theta = 31.7^\circ$$