

2013 Q10.

$$(a) \quad x^2 \frac{dy}{dx} = 7$$

$$\Rightarrow dy = \frac{7}{x^2} dx$$

$$\Rightarrow \int_1^y dy = 7 \int_7^{14} x^{-2} dx$$

$$\Rightarrow [y]_1^y = 7 \left[\frac{x^{-1}}{-1} \right]_7^{14}$$

$$\Rightarrow y - 1 = 7 \left(-\frac{1}{14} - -\frac{1}{7} \right)$$

$$\Rightarrow y - 1 = 7 \left(-\frac{1}{14} + \frac{1}{7} \right)$$

$$\Rightarrow y - 1 = \frac{1}{2} \frac{3}{28}$$

$$\Rightarrow y = 1.5$$

$$(b) (i) \quad \rightarrow a = (24t - 16) \text{ ms}^{-2}$$

0
t=0

$$\frac{dv}{dt} = 24t - 16 \Rightarrow dv = (24t - 16) dt$$

$$\Rightarrow \int_0^v dv = \int_0^3 (24t - 16) dt$$

$$\Rightarrow [v]_0^3 = \left[\frac{24t^2}{2} - 16t \right]_0^3$$

$$\Rightarrow v = 12(3)^2 - 16(3)$$

$$\Rightarrow v = 60 \text{ ms}^{-1}$$

$$[v = 12t^2 - 16t]$$

$$\frac{ds}{dt} = 12t^2 - 16t$$

$$\Rightarrow ds = (12t^2 - 16t) dt$$

$$\Rightarrow \int_0^s ds = \int_0^3 (12t^2 - 16t) dt$$

$$\Rightarrow s = \left[\frac{12t^3}{3} - \frac{16t^2}{2} \right]_0^3$$

$$\Rightarrow s = \left[4t^3 - 8t^2 \right]_0^3$$

$$\Rightarrow s = (4(3)^3 - 8(3)^2) - 0$$

$$\Rightarrow s = 36 \text{ m.}$$

$$(ii) \quad v = 12t^2 - 16t$$

$$80 = 12t^2 - 16t$$

$$\Rightarrow 12t^2 - 16t - 80 = 0$$

$$\Rightarrow 3t^2 - 4t - 20 = 0$$

$$\Rightarrow (3t - 10)(t + 2) = 0 \quad \Rightarrow t = \frac{10}{3} \text{ sec.}$$

$$(c) \quad \frac{dV}{dt} = -kV \quad \text{since } \frac{dV}{dt} \propto -V$$

$$\Rightarrow \frac{1}{V} dV = -k dt$$

$$\Rightarrow \int_V^{\frac{V}{2}} \frac{1}{V} \cdot dV = -k \int_0^t dt$$

$$\Rightarrow \left[\ln V \right]_V^{\frac{V}{2}} = -k \left[t \right]_0^t$$

$$\Rightarrow \left(\ln \frac{V}{2} - \ln V \right) = -k$$

$$\Rightarrow \ln \frac{\frac{V}{2}}{V} = -k$$

$$\Rightarrow \ln \frac{1}{2} = -k$$

$$\Rightarrow -0.693 = -k$$

$$\Rightarrow 0.693 = k$$

$$\int_V^{\frac{V}{5}} \frac{1}{V} dV = -0.693 \int_0^t dt$$

$$\Rightarrow \left[\ln V \right]_V^{\frac{V}{5}} = -0.693 \left[t \right]_0^t$$

$$\Rightarrow \ln \frac{V}{5} - \ln V = -0.693 t - 0$$

$$\Rightarrow \ln \frac{1}{5} = -0.693 t \Rightarrow t = 2.322 \text{ hrs.}$$

$$2.322 - 1 = 1.322 \text{ hrs} = 79.32 \text{ Mins}$$